B.Sc. Part III 7th Paper

(1)

DIFFERENTIAL EQUATIONS

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Method of variation of Parameters

where P, Q, R are either constants or functions of x.

Method First put $R = 0$

Now; find CF.

Let $CF = c_1 u + c_2 v$, where c_1, c_2 are constants.

Now $PI = u f(x) + v g(x)$

where $f(x) = -\int \frac{vR}{W}\, dx$

$g(x) = \int \frac{uR}{W}\, dx$

$W =$ Wronskian of u and $v = \begin{vmatrix} u & v \\ u_1 & v_1 \end{vmatrix}$

$= uv_1 - u_1 v$

Sums 1 solve $y_2 + n^2 y = \sec nx$

Soln comparing the given equation

$$\frac{d^2y}{dx^2} + 0\cdot\frac{dy}{dx} + n^2 y = \sec nx$$ with

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$, we have

$$P = 0,\ Q = n^2,\ R = \sec nx$$

Now put $R = 0$

$\Rightarrow$ for CF, $y_2 + n^2 y = 0$

$$\Rightarrow (D^2 + n^2)\,y = 0$$

$$\therefore D = \pm ni$$

$$\therefore CF = c_1 \cos nx + c_2 \sin nx.$$

Let $u = \cos nx$, $v = \sin nx$

$$\therefore W = \begin{vmatrix} u & v \\ u_1 & v_1 \end{vmatrix} = \begin{vmatrix} \cos nx & \sin nx \\ -n\sin nx & n\cos nx \end{vmatrix}$$

$$= n(\cos^2 nx + \sin^2 nx) = n$$

③

Now $f(x) = -\int \frac{VR}{W}\,dx$

$$\Rightarrow f(x) = -\int \frac{\sin nx \cdot \sec nx}{n}\,dx$$

$$= -\frac{1}{n}\int \tan nx\,dx = -\frac{1}{n^2}\log \sec nx$$

$$= \frac{1}{n^2}\log \cos nx$$

and $g(x) = \int \frac{UR}{W}\,dx$

$$\Rightarrow g(x) = \int \frac{\cos nx \cdot \sec nx}{n}\,dx = \frac{x}{n}$$

$\therefore$ $PI = uf(x) + vg(x)$

$$\Rightarrow PI = \cos nx \cdot \frac{1}{n^2}\log \cos nx + \sin nx \cdot \frac{x}{n}$$

Hence, the complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = c_1 \cos nx + c_2 \sin nx + \frac{1}{n^2}\cos nx \log \cos nx + \frac{x}{n}\sin nx.$$